Definition

The ℓ_2 - Norm

ℓ_∞ norm

for the vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = (x_1\ x_2\ x_3 \dots x_n)^T$

are defined by

$$\|x\|_2 = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2}$$

$$\|x\|_\infty = \max_{1 \le i \le n} |x_i|$$

Example

Determine the L_2 norm and L_∞ norm of the vector $x = (-1, 1, -2)^t$

Sol

$$\|x\|_2 = \sqrt{-1^2 + 1^2 + (-2)^2}$$

$$= \sqrt{6}$$

$$\|x\|_\infty = \max\{|-1|, |1|, |-2|\}.$$
$$= 2$$

Distance between vectors in $\mathbb{R}^n$

Definition — Exact solution

if $X = (x_1, x_2 - - x_n)^t$

$Y = (y_1 - - - y_n)^t$

$$\|X - Y\|_2 = \left(\sum_{i=1}^{n} (x_i - y_i) \right)^{1/2}$$

$$\|x - y\|_\infty = \max_{1 \le i \le n} |x_i - y_i|$$

Example

if a linear system, has an exact solution $X = (1,1,1)^t$ and the approximate solution is

$$\tilde{X} = (1.2001, 0.99991, 0.92538)$$

Determine the l_2 & L_∞ distances between the exact and approximation solution.

Sol

$$\|x-\tilde{x}\|_\infty = \max\{|1-1.2001|, |1-0.99991|, |1-0.92538|\}$$

$$= 0.2001$$

and

$$\|x-\tilde{x}\|_2 = \left[(1-1.2001)^2 + (1-0.99991)^2 + (1-0.92538)^2\right]^{1/2}$$

$$= 0.21356.$$

Diagonally Dominant Matrices.

for the system of equations.

$$Ax = b \quad \text{or} \quad [A|b]$$

the matrix A is Diagonal Dominant

if $\quad |a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$,

$1 < i < n$

Example.

for the system of eqs

$AX = b$

where

$$[A|b] = \left(\begin{array}{ccc|c} 5 & -1 & 1 & 10 \\ 2 & 4 & 0 & 12 \\ 1 & 1 & 5 & -1 \end{array}\right)$$

Is A is diagonal dominant?

Sol

$a_{11} = 5 > |-1| + |1|$

$a_{22} = 4 > |2| + |0|$

$a_{33} = 5 > |1| + |1|$

then

A is diagonally dominant

end.